# BINARY STAR ORBITS. III. REVISITING THE REMARKABLE CASE OF TWEEDLEDUM AND TWEEDLEDEE

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#### **ABSTRACT**

Two of the most challenging objects for optical interferometry in the middle of the last century were the close components (FIN 332) of the wide visual binary STF2375 (= WDS 18455+0530 = HIP 92027 = ADS 11640). Each component of the wide pair was found to have subcomponents of approximately the same magnitude, position angle, and separation and, hence, were designated by the tongue-in-cheek monikers "Tweedledum and Tweedledee" by the great visual interferometrist William S. Finsen in 1953. They were later included in a list of "Double Stars that Vex the Observer" by W.H. van den Bos in 1958. While speckle interferometry has reaped a rich harvest investigating the close inteferometric binaries of Finsen, the "Tweedles" have continued to both fascinate and exasperate due to both the great similarity of the close pairs and the inherent 180° ambiguity associated with interferometry. Detailed analysis of all published observations of the system has revealed several errors which are here corrected, allowing for determination of these orbital elements which resolve the quadrant ambiguity. A unique software filter was developed which allowed subarrays from archival ICCD speckle data from 1982 to be re-reduced. Those data, combined with new and unpublished observations obtained in 2001–2009 from NOAO 4 m telescopes, the Mount Wilson 100 inch telescope and the Naval Observatory Flagstaff Station 61 inch telescope as well as high-quality unresolved measures all allow for the correct orbits to be determined. Co-planarity of the multiple system is also investigated.

Key words: binaries: general – binaries: visual – stars: individual (HR 7048) – techniques: interferometric

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# 1. THE DISCOVERY AND EARLY MEASURES OF TWEEDLEDUM AND TWEEDLEDEE

The bright star HR 7048 [= HD 173495 = HIP 92027 =ADS 11640 = STF2375,  $(\alpha,\delta) = 18^{h}45^{m}28^{s}4 + 05^{\circ}30'00''$ (2000)] was first recognized as a double by F.G.W. Struve in 1825 (Struve 1837). Since that time the system has been well observed by many double star astronomers, and has probably been most useful for those wishing to characterize or calibrate their observational systematics, as the motion has long been recognized as quite slow. As early as the start of the last century, Burnham (1906) in his double star catalog (where this object is listed at 8776) noted "no change in distance, and but little, if any, in the angle." Almost a century later we have seen a cumulative change of only 12° and 0".3 since the discovery epoch. The first indication that this system might be more than just a slowly moving pair came in a compelling note to Aitken's (1932) catalog which stated that the "radial velocity of A is variable with a range of 99 km s<sup>-1</sup>," citing no less an authority than Plaskett et al. (1921) as the source. The source of the variability—and the system's interest—was discovered by William Finsen some two decades later.

After experimenting with different interferometer designs, Finsen (1964a) had constructed an eyepiece interferometer, where the observer visually measured interferometric fringe visibilities, then calculated position angles (P.A.s) and separations. This instrument, as with other interferometric techniques, was best suited to brighter stars and therefore, a program commenced

to investigate the duplicity of all 8117 stars brighter than magnitude 6.5 with  $+20^{\circ} < \delta < -75^{\circ}$ . In addition to measuring many thousands of known systems, application of this new technique starting in 1951 (Finsen 1951) led to the discovery of 79 new pairs (Mason et al. 2001) almost all of which are close and astrophysically interesting due to their rapid motion.

However, upon turning to the wide components of the Struve pair, Finsen was initially surprised and confused. His first observations were rather vexing, as he reported in his article "A Case of Tweedledum and Tweedledee" (Finsen 1953, p. 86, 1954, p. 41):

"The two pairs are remarkably similar; in fact, the simultaneous disappearance of both sets of fringes gave rise to considerable dismay till careful checking showed there was nothing wrong with the instrument."

Apparently, Finsen was quite careful and did not trust his result fully until independently confirmed and measured (albeit crudely) with a micrometer by van den Bos (1956) two nights later. These measures were also quoted in Finsen's discovery paper (1953, 1954). The systems were observed and reported on a fairly regular basis in the 1950s and early 1960s, with eyepiece interferometry by Finsen and with micrometry on large refractors and reflectors by van den Bos, van Biesbroeck, and Muller (see Tables 1 and 2 for all measures and references). Both systems then disappeared from sight (like the Cheshire Cat?) as reported by Finsen (1965, 1967, 1969), van den Bos (1963b), and Worley (1972), although Walker (1969) listed a measure for Ba,Bb (then designated CD) obtained in 1966.

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<sup>&</sup>lt;sup>4</sup> Tweedledum and Tweedledee are nursery rhyme characters whose names first appeared in an epigram by J. Byrom (1692–1763). They are best known as a pair of identical twins reciting these rhymes in Lewis Carroll's *Through the Looking Glass and What Alice Found There* (Dodgson 1871).

# **Report Documentation Page**

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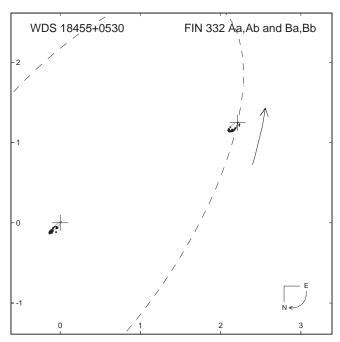
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13. SUPPLEMENTARY NOTES

14. ABSTRACT

Two of the most challenging objects for optical interferometry in the middle of the last century were the close components (FIN 332) of thewide visual binary STF2375 (=WDS18455+0530=HIP 92027=ADS 11640). Each component of thewide pairwas found to have subcomponents of approximately the samemagnitude, position angle and separation and, hence, were designated by the tongue-in-cheek monikers ?Tweedledum and Tweedledee? by the great visual interferometristWilliam S. Finsen in 1953. They were later included in a list of ?Double Stars that Vex the Observer? by W.H. van den Bos in 1958. While speckle interferometry has reaped a rich harvest investigating the close inteferometric binaries of Finsen, the ?Tweedles? have continued to both fascinate and exasperate due to both the great similarity of the close pairs and the inherent 180◦ ambiguity associated with interferometry. Detailed analysis of all published observations of the system has revealed several errors which are here corrected, allowing for determination of these orbital elements which resolve the quadrant ambiguity. Aunique software filterwas developed which allowed subarrays from archival ICCD speckle data from 1982 to be re-reduced. Those data, combined with new and unpublished observations obtained in 2001?2009 from NOAO 4 m telescopes, the Mount Wilson 100 inch telescope and the Naval Observatory Flagstaff Station 61 inch telescope as well as high-quality unresolved measures all allow for the correct orbits to be determined. Co-planarity of the multiple system is also investigated.

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**Figure 1.** "Ferris Wheel" plot of FIN 332 Aa, Ab and Ba, Bb, shown to the same scale as that of the wide pair, STF2375. These are the orbits of Figures 5 and 6. In this plot, the relative positions of STF2375 A and B are fixed and the dashed curve is indicative of the pair's orbital motion (although it has only moved 12° since its discovery in 1825, so no believable orbital elements can be determined). The amount and direction of motion of the AB pair over the past 185 years are indicated by the thick curved arrow. The arrow at lower right indicates the direction of motion of both close pairs, which is opposite to that of the wide pair. Scales are in arcseconds.

Throughout this time STF2375 retained considerable interest and was among six systems described in some detail by van den Bos (1958a, p. 7) in his article "Double Stars that Vex the Observer", where he elaborated a bit upon Finsen's discovery:

"...When inspecting ADS 11640, Finsen was startled to see the fringes on both components of the Struve pair disappear simultaneously when rotating the interferometer. He suspected, at first, that something had gone wrong with the instrument, but other stars showed nothing abnormal and it turned out that he had indeed found, not fraternal but identical twins, for which he applied the nicknames 'Tweedledum and Tweedledee.'

I have recently measured this object with the Lick 36 inch refractor which clearly separates the two close pairs and the appearance is astonishing. Apart from the fact that Tweedledee ... is slightly fainter than Tweedledum ..., I can see no difference between the two ..."

#### 2. GETTING TOO CLOSE TO RESOLVE

From the first work of John Herschel (1847), through the large survey of Rossiter (1955) and the work of Finsen and van den Bos at Union and (later) Republic Observatory, double star work at the Cape could be characterized in one of two ways: excellent or inactive. The disappearance of the Tweedles in retrospect seemed to portend a period of benign neglect at the Cape, as van den Bos left for the United States and Finsen approached retirement with some trepidation, as he wrote to Charles Worley (1968, private communication):

"...There is a move afoot to give first and absolute priority to a programme of planetary photography, to the distress of van den Bos and myself... and this seems likely to ring the death knell of our long record of intensive double star observing. I found it impossible to explain to people with no experience of double star observing of the demands it places on the observer's skill, enthusiasm and energy to relegate that to a second priority time-filling role is very discouraging, to say the least of it, and may very well kill it stone dead. Time will show."

As Finsen predicted, double star astronomy in South Africa saw a definite downturn after his retirement. Fortunately, some 10 years after the demise of eyepiece interferometry the technique of speckle interferometry was developed. In the late 1970s, one of the authors (H.A.M.) began a healthy correspondence with Finsen just as his speckle program was getting started, regarding objects which would be suitable for speckle interferometry. Finsen's continued interest in this pair is apparent in his letter of 1977 (W. S. Finsen 1977, private communication):

"... I was reminded of the quadruple that I have dubbed 'Tweedledum and Tweedledee'... Have you got this on your programme? It would be fun if you could follow it up and eventually do the orbits. These 'identical twins' caused me much agony of mind before I was prepared to accept their duplicity as real. I measured them regularly until 1963 when both became too close to measure without much change in position angles."

# 3. SPECKLE INTERFEROMETRY: THE REAPPEARANCE OF THE TWEEDLES

Due to their spatially close nature, many of the systems first resolved by Finsen have orbital periods of less than 50 years. Thus, speckle interferometry became a mature technique at an optimal time for orbital analysis of many of Finsen's discoveries; observation of the Finsen stars was therefore given high priority in the early years of this technique. Early results of those efforts include orbital analyses of FIN 342 (McAlister et al. 1988), FIN 312 (Hartkopf et al. 1989), FIN 331, 325, 350, 381 (Hartkopf et al. 1996), FIN 347 (Mason et al. 1996), FIN 359 (Mason 1997), FIN 47, and 328 (Mason et al. 1999).

FIN 332 Aa, Ab and Ba, Bb were both recovered by speckle interferometry in 1976 and continued to be observed on numerous occasions by this and other interferometric techniques (see Tables 1 and 2, and reference quoted therein).

Figure 1 presents a demonstration of the similarity in spatial characteristics of the Tweedles in a "Ferris Wheel" plot. In this diagram, the two pairs are shown relative to each other and to the same scale and orientation. The small ellipse in the lower left is the calculated orbit of FIN 332 Aa, Ab while the one in the upper right is Ba, Bb. The large dashed ellipse is an indication of the motion of the wider pair, although given the very small coverage of the orbit it is only present to give an idea of the relative scales of the orbits. The axes are in arcseconds. The orbits of the close pairs are described in Section 5.1.

### 4. MEASURES OF THE TWEEDLES

Tables 1 and 2 present the observations of FIN 332 Aa, Ab and Ba, Bb, respectively. Columns 1–4 contain data specific to the observation: the epoch of observation (expressed as fractional Besselian year), position angle (P.A., in degrees), separation (in arcseconds), and number of measures comprising this mean published position. Note that the P.A. has not been corrected for precession and is thus based on the equinox for the epoch

Table 1 Measurements of FIN 332Aa, Ab

				Wicasarcinei	113 01 1 11 1 3327 10	1, 710		
Epoch	θ (0)	ρ (")	n	<i>O−C</i> (○)	O-C (")	Method	Reference	Notes
1953.73	316.5	0.153	5	3.0	-0.014	E	Finsen 1953	1
1953.75	315.7	0.155	1	2.2	-0.014 $-0.017$	M	van den Bos 1956	1
1954.68	302.7	0.158	4	-10.2	-0.007	E	Finsen 1956	1
1955.72	309.8	0.144	3	-2.3	-0.017	E	Finsen 1956	1
1957.39	311.9	0.15	4	1.2	-0.002	M	van den Bos 1958b	1
1957.76	314.7	0.144	1	4.3	-0.005	E	Finsen 1959	1
1957.89	314.6	0.13	4	4.3	-0.018	M	van Biesbroeck 1960	1
1958.54	311.6	0.15	3	1.9	0.008	M	van den Bos 1960	1
1959.72	302.9	0.131	3	-5.6	0.000	Е	Finsen 1960	1
1960.564	318.2	0.14	6	10.8	0.019	M	van Biesbroeck 1965	1
1960.72	298.9	0.137	1	-8.3	0.018	Е	Finsen 1961	1
1961.57	312.6	0.11	5	6.7	0.003	M	van den Bos 1962	1
1961.73	297.8	0.112	3	-7.8	0.008	E	Finsen 1962	1
1962.51	314.7	0.10	4	10.7	0.008	M	van den Bos 1963a	1
1962.72	309.2	0.114	5	5.7	0.026	E	Finsen 1963	1
1963.38	unre	solved	1	(301.5)	(0.076)	M	van den Bos 1963b	1,2
1963.728	313.0	0.106	4	12.8	0.037	E	Finsen 1964b	1
1964.726	unre	solved	1	(294.1)	(0.046)	E	Finsen 1965	1,2
1966.758	unre	solved	1	(157.0)	(0.016)	E	Finsen 1967	1,2
1968.791	unre	solved	1	(334.2)	(0.040)	E	Finsen 1969	1,2
1971.531	307.0	0.15	1	-15.2	0.046	M	Walker 1972	1
1975.48	316.9	0.12	3	-0.6	-0.031	M	Heintz 1978	1
1976.2992	318.1	0.143	1	1.2	-0.014	Sp	McAlister 1978	
1976.3702	318.5	0.149	1	1.7	-0.008	Sp	McAlister & Hendry 1982a	
1976.3728	320.5	0.164	1	3.7	0.007	Sp	McAlister & Hendry 1982a	
1976.4549	317.4	0.161	1	0.6	0.004	Sp	McAlister 1978	
1977.3340	316.9	0.158	1	0.8	-0.004	Sp	McAlister & Hendry 1982a	
1977.4815	316.4	0.162	1	0.4	-0.000	Sp	McAlister 1979	
1977.4871	316.2	0.164	1	0.2	0.002	Sp	McAlister 1979	
1977.521	312.9	0.18	3	-2.9	-0.017	M	Walker 1985	1
1977.6400	315.9	0.175	1	0.0	0.012	Sp	McAlister & Hendry 1982a	
1978.5410	316.2	0.170	1	1.0	0.004	Sp	McAlister & Fekel 1980	
1978.6147	316.6	0.170	1	1.4	0.004	Sp	McAlister & Fekel 1980	
1979.3601	314.0	0.170	1	-0.6	0.003	Sp	McAlister & Hendry 1982b	
1979.5321	313.2	0.151	1	-1.3	-0.016	Sp	McAlister & Hendry 1982b	
1979.7725	312.5	0.166	1	-1.8	-0.001	Sp	McAlister & Hendry 1982b	
1980.4769	311.4	0.173	1	-2.4	0.006	Sp	McAlister et al. 1983	
1980.4794	314.7	0.169	1	0.9	0.002	Sp	McAlister & Hartkopf 1984	
1980.7173	311.0	0.159	1	-2.7	-0.008	Sp	McAlister et al. 1983	
1980.7199	311.8	0.169	1	-1.9	0.002	Sp P	McAlister et al. 1983	1
1981.356	313.2	0.186	1	0.1	0.020		Tokovinin 1982	1
1982.5029	316.4	0.165	1	4.0 3.0	0.003 $-0.002$	Sc	This paper	3
1982.5248 1982.7650	315.2	0.160	1		0.002	Sc	Fu et al. 1997 This paper	1
1982.7630	313.0	0.162 0.157	2 1	0.8 0.7	0.001	Sc Sc	McAlister et al. 1987a	3 4
1983.4203	312.3 312.4	0.137	1	1.5	-0.001	Sc	Hartkopf et al. 2000	5
1984.783	335.9	0.147	1	25.5	-0.003 $-0.022$	P	Tokovinin & Ismailov 1988	1
1985.4816	310.7	0.127	1	0.8	-0.022 $-0.004$	Sc	McAlister et al. 1987a	4
1985.5231	310.7	0.139	1	0.5	-0.004 $-0.001$	Sc	McAlister et al. 1987b	4
1985.7440	318.3	0.142	1	8.8	-0.001 $-0.004$	P	Tokovinin & Ismailov 1988	1
1985.8424	309.2	0.137	1	-0.3	0.000	Sc	McAlister et al. 1987a	4
1987.7618	309.2	0.140	1	2.0	-0.001	Sc	McAlister et al. 1989	4
1988.6655	305.1	0.117	1	-0.7	0.001	Sc	McAlister et al. 1990	-
1990.2734	305.6	0.083	1	3.6	0.005	Sc	Hartkopf et al. 1992	
1991.2500		solved	1	(297.8)	(0.058)	H	ESA 1997	2,6
1992.3105	ume	< 0.038	1	(286.8)	(0.032)	Sc	This paper	2,7
1996.3214	318.2	0.067	1	-11.4	0.013	Sc	This paper	8
1996.6930	333.0	0.071	1	5.5	0.007	Sc	Hartkopf et al. 2000	O
1990.0930	326.0	0.071	1	1.1	0.007	S	Balega et al. 1999	
1997.4630	328.2	0.082	1	3.5	-0.003	Sc	This paper	9
2001.4988	319.4	0.079	1	1.0	0.003	S*	This paper This paper	10
2001.5697	318.2	0.136	1	-0.1	-0.002	S*	This paper	11
2001.3037	308.2	0.150	1	-6.7	0.002	S*	This paper This paper	10
2005.8052	315.3	0.165	1	0.7	-0.002	S* S*	This paper	11
2006.5640	316.1	0.165	1	1.8	-0.002 $-0.002$	S* S*	This paper This paper	9
2000.2040	310.1	0.103	1	1.0	0.002	D×.	ims paper	,

Table 1 (Continued)

Epoch	θ	ρ	n	O-C	O-C	Method	Reference	Notes
	(0)	(")		(0)	(")			
2007.3174	313.6	0.170	1	-0.2	0.003	S⋆	This paper	9
2007.5879	310.5	0.179	2	-3.1	0.012	S⋆	This paper	10
2007.8010	311.6	0.168	1	-1.9	0.001	S⋆	This paper	9
2008.4529	313.0	0.168	4	-0.1	0.003	S⋆	This paper	10
2008.5371	314.4	0.168	2	1.5	0.003	S	Tokovinin et al. 2010	12
2008.7721	314.8	0.159	1	2.0	-0.006	S	Tokovinin et al. 2010	13
2008.8712	316.9	0.192	2	4.2	0.028	S⋆	This paper	1, 14
2009.2607	312.3	0.162	2	-0.1	-0.001	S	Tokovinin et al. 2010	12

Notes. Methods: E, eyepiece interferometer; H, Hipparcos observation; M, micrometer; P, phase grating interferometer; S, speckle interferometer; Sp, photographic speckle camera of McAlister (1977); and Sc, ICCD speckle camera of McAlister et al. (1987a); S⋆, USNO speckle camera of Mason et al. (2009).

- 1: measure not used in new orbit solution.
- 2: here Columns 4 and 5 give the predicted position of the secondary relative to the primary.
- 3: measure obtained by re-reduction of CCD subarray. See Section 4.4.
- 4: the original calibration was corrected in McAlister et al. (1989) and this corrected measure first published in McAlister & Hartkopf (1988).
- 5: re-reduction of data yielded improved signal-to-noise ratio and allowed this measure to be made.
- 6: no measure of this subsystem was published in the Hipparcos Catalogue.
- 7: the other pair, Ba, Bb (see Table 2) was measured at this time, so this is judged to be a reliable null detection.
- 8: measure inadvertently left out of Hartkopf et al. (2000).
- 9: observation made on Mount Wilson 100".
- 10: observation made on KPNO 4 m.
- 11: observation made on CTIO 4 m.
- 12:  $\Delta m$  is  $0.9 \pm 0.4$  in Strömgren y.
- 13:  $\Delta m$  is 1.3 in H $\alpha$ .
- 14: Observation made on NOFS 61".

of observation. When the pair is unresolved the lower limit on separation is given in Column 3 if published or determined here. Columns 5 and 6 give O-C orbit residuals (in  $\theta$  and  $\rho$ ) to the orbits presented in Section 5.1. When the components are unresolved, the O-C columns (Columns 5 and 6) now give, in parentheses, the position predicted by these orbits. The method of observation is indicated in Column 7, while the reference to the measure is in Column 8. The Center for High Angular Resolution Astronomy (CHARA) photographic speckle camera was less sensitive than the ICCD system, as seen by the small number of measures of the fainter Ba, Bb pair (N = 8) versus Aa, Ab (N = 17). Finally, Column 9 is reserved for the many notes to the measures. In addition to quadrant flips indicated by the correct determination of this previously ambiguous characteristic, there are also other cases where the originally published measures have been corrected. These are described in Section 4.1.

A representation of the similarity of measurements of these systems to each other is presented in Figures 2(a) and 2(b). Note that the predicted separation and P.A. differences (assuming an arbitrary quadrant, i.e.,  $\pm 180^{\circ}$ ) are usually quite small, especially at the time of the discovery and during the first phase of resolutions (Section 1) where  $\Delta \rho < 0''.04$  and  $\Delta \theta < 3^{\circ}$ . The two curves and shaded regions, representing the orbital solutions, are presented below.

Table 3 provides measures, contemporaneous with those new measures presented here, of the wider AB parent pair, STF2375. Columns 1–4 are as Tables 1 and 2. Column 5 provides the method while Column 6 the notes. In this case, the notes simply indicate which telescope was used.

#### 4.1. Corrections to Published Measures

A total of seven measures (Aa, Ab: 4, Ba, Bb: 3) were initially published by CHARA using a preliminary calibration.

This calibration was corrected in McAlister et al. (1989) and the corrected measures first appeared in McAlister & Hartkopf (1988); the corrected measures are listed here. While the very small  $\Delta m$  of Aa relative to Ab and of Ba relative to Bb presents one set of unique problems, another is the small (but easily measurable) difference between the different components in the wide STF2375 system. Normally, the field of view is such that it is possible to observe both pairs; however, problems with analysis of the complex system (see Section 4.4 below), led the CHARA Collaboration to observe this system under high magnification so that only one pair was resolved at a time. In this case, a measure of the Ba, Bb pair (Hartkopf et al. 1997) was incorrectly assigned to Aa, Ab.

Among the most difficult sets of observations to sort out were the 1984 and 1985 observations of the Ba, Bb pair made by Tokovinin & Ismailov (1988). After investigating numerous possible quadrant flips and/or identification errors for these measures and incorporating the unresolved measures in the analysis the two measures did not fit any orbital analysis obeying Kepler's laws. The first author was consulted, and it is possibly most instructive at this point to quote directly from his response (A. A. Tokovinin 2001, private communication):

"I observed it myself at the 1 m telescope in 1981, 1984, 1985 with the phase grating interferometer. Both close pairs fell within the focal aperture, so for this object I had to de-center and hide either Aa, Ab or Ba, Bb (in your notation) behind the diaphragm, to get the visibilities of the remaining pair. It is very unlikely that I misidentified the close pairs ... For pairs of this separation, the curve of visibility versus  $\theta$  has two rather similar maxima. Apparently, in reducing the 1984–1985 Ba, Bb data I took the wrong one: this changes the position angle by roughly  $90^{\circ}$  and gives similar, but wrong separation. The choice of the 'correct' maximum was often guided by the previous

# MASON, HARTKOPF, & MCALISTER

Table 2 Measurements of FIN 332Ba, Bb

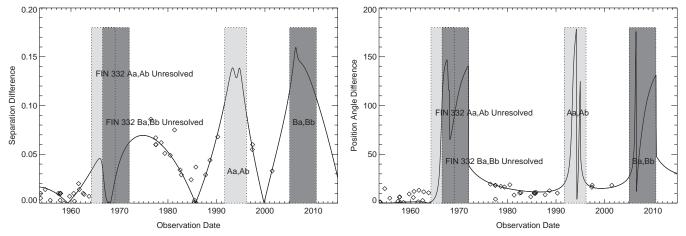
Epoch	$\theta$	$\rho$	n	<i>O</i> – <i>C</i>	O-C	Method	Reference	Notes
	(0)	(")		(0)	(")			
1953.73	315.3	0.148	5	1.3	-0.002	Е	Finsen 1953	1
1953.74	315.1	0.14	1	1.1	-0.010	M	van den Bos 1956	1
1954.68	317.6	0.144	4	4.6	-0.005	E	Finsen 1956	1
1955.72	314.9	0.141	3	3.0	-0.007	Е	Finsen 1956	1
1957.39	310.0	0.16	4	-0.1	0.017	M	van den Bos 1958b	1
1957.73	300.	0.15	2	-9.7	0.008	M	Muller 1958	1
1957.73	306.	0.14	1	-3.7	-0.002	M	Muller 1958	1
1957.76	308.3	0.147	1	-1.4	0.005	Е	Finsen 1959	1
1957.89	313.8	0.12	4	4.3	-0.021	M	van Biesbroeck 1960	1
1958.54	312.3	0.15	3	3.6	0.011	M	van den Bos 1960	1
1959.72	312.3	0.124	3	5.1	-0.009	Е	Finsen 1960	1
1960.564	312.4	0.13	6	5.8	0.003	M	van Biesbroeck 1965	1
1960.72	310.9	0.139	1	5.1	0.013	Е	Finsen 1961	1
1961.57	311.2	0.13	5	6.7	0.010	M	van den Bos 1962	1
1961.73	311.0	0.126	3	6.7	0.007	Е	Finsen 1962	1
1962.51	312.0	0.11	4	9.1	-0.002	M	van den Bos 1963a	1
1962.72	320.8	0.123	5	18.3	0.014	E	Finsen 1963	1
1963.38		solved	1	(301.1)	(0.102)	M	van den Bos 1963b	1,2
1963.728	323.6	0.113	4	23.3	0.015	Е	Finsen 1964b	1
1964.726		solved	1	(297.5)	(0.084)	Е	Finsen 1965	1,2
1966.436	276.7	0.26	1	-11.7	0.210	M	Walker 1969	1
1966.758	unre	solved	1	(284.8)	(0.041)	E	Finsen 1967	1,2
1968.791		solved	1	(72.1)	(0.023)	E	Finsen 1969	1,2
1971.504	90.0	0.15	1	84.9	0.105	M	Walker 1972	1
1976.4549	336.9	0.075	1	-0.3	-0.015	Sp	McAlister 1978	
1977.4815	334.6	0.095	1	0.2	-0.003	Sp	McAlister 1979	
1977.4870	334.6	0.104	1	0.2	0.006	Sp	McAlister 1979	
1977.521	317.0	0.12	3	-17.2	0.022	M	Walker 1985	1
1978.6147	333.8	0.108	1	1.9	0.002	Sp	McAlister & Fekel 1980	
1979.3601	330.5	0.119	1	0.1	0.008	Sp	McAlister & Hendry 1982b	
1980.4769	330.2	0.124	1	1.8	0.006	Sp	McAlister et al. 1983	
1981.356	321.5	0.111	1	-5.4	-0.012	P	Tokovinin 1982	1
1981.4681	327.2	0.114	1	0.4	-0.009	Sp	McAlister et al. 1984	
1981.6975	325.0	0.120	1	-1.5	-0.005	Sp	McAlister et al. 1984	
1982.5029	326.8	0.131	1	1.5	0.002	Sc	This Paper	3
1982.7650	323.7	0.133	1	-1.2	0.003	Sc	This Paper	3
1984.783	329.4	0.103	1	7.2	-0.035	P	Tokovinin & Ismailov 1988	1,4
1985.4816	321.3	0.140	1	-0.1	-0.001	Sc	McAlister et al. 1987a	5
1985.5231	320.4	0.139	1	-1.0	-0.002	Sc	McAlister et al. 1987b	5
1985.7440	308.3	0.100	1	-12.8	-0.042	P	Tokovinin & Ismailov 1988	1,4
1985.8424	320.4	0.140	1	-0.6	-0.002	Sc	McAlister et al. 1987a	5
1987.7618	317.8	0.146	1	-1.1	-0.001	Sc	McAlister et al. 1989	
1988.6655	317.8	0.151	1	-0.1	0.003	Sc	McAlister et al. 1990	
1990.2734	315.9	0.151	1	-0.3	0.001	Sc	Hartkopf et al. 1992	
1991.2500	308.	0.16	0	-7.1	-0.009	Н	ESA 1997	1,6
1992.3105	314.5	0.153	1	0.4	0.003	Sc	Hartkopf et al. 1994	
1995.6008	306.1	0.136	1	-4.5	-0.009	Sc	Hartkopf et al. 1997	7
1995.6061	311.8	0.141	1	1.2	-0.004	Sc	Hartkopf et al. 2000	
1996.3215	310.7	0.141	1	1.0	-0.001	Sc	Hartkopf et al. 2000	
1996.3270	310.1	0.142	1	0.4	-0.000	Sc	Hartkopf et al. 2000	
1996.7012	307.1	0.139	1	-2.2	-0.002	Sc	Hartkopf et al. 2000	
1997.3945	309.7	0.137	1	1.2	-0.001	S	Balega et al. 1999	
1997.4630	309.8	0.139	1	1.4	0.002	Sc	This paper	8
2001.4988	301.1	0.111	3	-1.1	0.003	S⋆	This paper	9
2001.5697	304.7	0.105	1	2.6	-0.002	S*	This paper	10
2005.8652		< 0.038	1	(273.7)	(0.024)	S*	This paper	2,9,11
2006.2001		< 0.038	1	(243.3)	(0.011)	S*	This paper	2,10,11
2006.5640		< 0.060	1	(127.2)	(0.013)	S⋆	This paper	2,11,12
2007.3174		< 0.060	1	(76.7)	(0.023)	S*	This paper	2,11,12
2007.5879		< 0.038	1	(65.0)	(0.024)	S*	This paper	2,9,11
		< 0.060	1	(56.7)	(0.025)	S*	This paper	2,11,12
2007.8010								
2007.8010 2008.4615	28.8	0.049	2	-6.7	0.020	S⋆	This paper	9

# Table 2 (Continued)

Epoch	θ (ο)	ρ (")	n	<i>O−C</i> (⋄)	O-C (")	Method	Reference	Notes
2008.8658		<0.098	1	(25.6)	(0.033)	S*	This paper	2,14
2009.2607		<0.050	1	(17.8)	(0.036)	S	Tokovinin et al. 2010	2,15

Notes. Methods: E, eyepiece interferometer; H, Hipparcos observation; M, micrometer; P, phase grating interferometer; S, speckle interferometer; Sp, photographic speckle camera of McAlister (1977); Sc, ICCD speckle camera of McAlister et al. (1987a); and S★, USNO speckle camera of Mason et al. (2009).

- 1: measure not used in new orbit solution.
- 2: here Columns 4 and 5 give the predicted position of the secondary relative to the primary.
- 3: measure obtained by re-reduction of CCD subarray. See Section 4.4.
- 4: published P.A. was 59.4, and 38.3 and given zero weight in orbit determination. See Section 4.1.
- 5: the original calibration was corrected in McAlister et al. (1989) and this corrected measure first published in McAlister & Hartkopf (1988).
- 6: the  $H_p$  magnitude difference is  $0.76 \pm 0.15$ .
- 7: assigned in error to Aa, Ab in Hartkopf et al. (1997).
- 8: observation made on Mount Wilson 100".
- 9: observation made on KPNO 4 m.
- 10: observation made on CTIO 4 m.
- 11: the other pair, Aa, Ab (see Table 1) was measured at this time, so this is judged to be a reliable null detection.
- 12: observation made on Mount Wilson 100". Not plotted in Figure 6.
- 13:  $\Delta m$  is 0.5 in Strömgren y.
- 14: observation made on NOFS 61". Not plotted in Figure 6.
- 15: observation obtained on the SOAR 4.2 m telescope. While Ba, Bb was previously resolved when it was closer according to Tokovinin: "Bab could be partially resolvable, but in the AD (atmospheric dispersion) direction. Fits do not converge, so it remains unresolved. The AD was 3.2 pixels, so if the pair was under 50 mas or so, the negative result could be explained."



**Figure 2.** Difference in separation and P.A. between FIN 332 Aa, Ab and Ba, Bb. The solid curve indicates the predicted difference based on the orbits of Section 5.1. The diamonds represent the differences between measures of Aa, Ab and Ba, Bb when they were made at the same time by the same observer. The lighter shaded areas from 1964.2 to 1969.1 and 1991.7 to 1996.2 are dates when Aa, Ab is predicted to be closer than 0″.05. The darker shaded areas from 1966.5 to 1972.0 and 2005.1 to 2010.6 indicate the range of dates when the orbit predicts Ba, Bb to be closer than 0″.05.

measurements, and, apparently, in this case was wrong! So, the data on Aa, Ab as measured in 1984–1985 must be still valid, and not attributed to Ba, Bb. Measurement error, however, can still be too big, compared to the 4 m speckle, because it is a difficult object, it was de-centered, etc..."

Given this, it is not surprising, despite the  $90^{\circ}$  adjustment, that these measures had residuals judged too large by the orbit calculation.

# 4.2. Hipparcos

The Hipparcos satellite (ESA 1997) observed STF2375 and resolved the wider AB pair and the Ba, Bb pair at the calculated date of 1991.25. The Aa, Ab pair was not resolved. Due to the substantial orbital motion of the Ba, Bb pair during the course of the Hipparcos mission, the quality of this measure may be somewhat degraded.

While all components have the same parallax and proper motions [ $\pi = 4.60 \pm 1.10$  mas,  $\mu_{\alpha} = 15.54 \pm 1.07$  mas,  $\mu_{\delta} = 1.96 \pm 0.86$  mas (ESA 1997)  $\pi = 5.30 \pm 0.85$  mas,  $\mu_{\alpha} = 14.32 \pm 1.04 \text{ mas}, \, \mu_{\delta} = 0.16 \pm 0.87 \text{ mas} \, (\text{van Leeuwen})$ 2007)], the errors are probably larger than this, possibly by as much as 50% (see Urban et al. 2000). While the reasons for the error underestimation may be complex, it is likely that long-term motion of wide pairs may not be fully characterized in the few year Hipparcos solution. The Tycho-2 (Høg et al. 2000a, 2000b) proper motion, which is possibly more accurate for long period doubles like AB, is  $\mu_{\alpha} = 9.4 \pm 2.0$  mas,  $\mu_{\delta} = -2.5 \pm 1.9$ mas. Normally a system this bright would have many historical measures to improve the proper motion. However, as the AB system was judged a close pair for transit circles it was left off many of these programs and only three historical measures, Albany 10, AGK2, and AGK3, were used in the Tycho-2 proper motion determination; all of these were photocenter

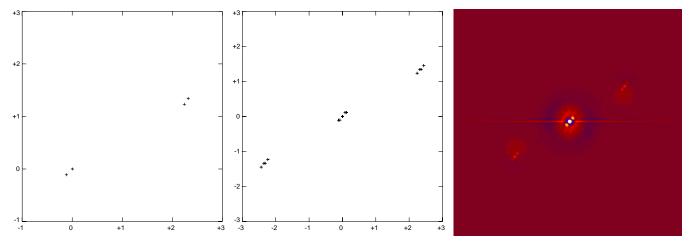


Figure 3. Panel (a) is a representation of the geometry of the system at 1982.7650 based on the new measures of the 1982 data. The scales are in arcseconds. The origin is the location of the Aa component in relative astrometry space. Panel (b) is the autocorrelation of panel (a). Note the four visible "double + signs" which represent the blends described in Section 4.4. The central peak is the zeroth-order autocorrelation spike. The gray circles, barely visible at this scale, are 0''.030 in diameter to indicate regions where detail cannot be seen due to the resolution capabilities of the telescope. Panel (c) is a digitization of the 1982.7650 data with the blended images. While some of the peaks are quite faint, all nine visible peaks of panel (b) are seen here.

(A color version of this figure is available in the online journal.)

Table 3 Measurements of STF2375AB

Epoch	θ	ρ	n	Method	Notes
-r	(0)	(")			
1997.4657	119.6	2.590	1	Sc	1
2004.2019	122.9	2.496	1	S⋆	2
2006.1974	120.1	2.549	1	S⋆	3
2006.5640	119.7	2.512	1	S⋆	1
2007.3174	119.6	2.484	1	S⋆	1
2007.5879	118.2	2.537	1	S⋆	4
2007.8010	118.2	2.537	1	S⋆	1
2008.4569	119.5	2.504	3	S⋆	4
2008.8549	119.1	2.618	3	S⋆	2
2008.8712	119.1	2.569	3	S⋆	2

Notes. Methods: Sc, ICCD speckle camera of McAlister et al. (1987a) and S⋆, USNO speckle camera of Mason et al. (2009).

- 1: observation made on Mount Wilson 100".
- 2: observation made on NOFS 61".
- 3: observation made on CTIO 4 m.
- 4: observation made on KPNO 4 m.

observations (S. E. Urban 2002, private communication). Of greater concern is the large  $\Delta m$  assigned by Hipparcos to the Ba, Bb pair of  $H_p = 0.76$  mag. Although the errors are large  $(Ba = 7.192 \pm 0.155 \text{ mag}, Bb = 7.952 \pm 0.315 \text{ mag})$ , it is certainly difficult to reconcile this large  $\Delta m$  with the many visual estimates. It may be the result of too many free parameters for four physically related components, even though only three were resolved.

### 4.3. The Magnitude Difference

Based on published pre-speckle magnitude difference estimates, Aa, Ab has a mean  $\Delta m$  of  $0.008 \pm 0.119$  while Ba, Bb has a mean of  $0.043 \pm 0.232$ . This quadrant ambiguity can result in two consistent results: one solution is of period P and high eccentricity and, contrariwise (as one of the Tweedles might say), another solution is of period 2P and low eccentricity. While we have contemporary measures of  $\Delta m$  (see the notes to Tables 1 and 2) which are larger, this only gives the absolute orientation at a single epoch; establishing which orbit is correct requires information from data at either end of the long period solution.

The quadrant analysis of Bagnuolo et al. (1992) was successfully used on FIN 342 (McAlister et al. 1988), another binary of small  $\Delta m$ , and this method was utilized here to definitely establish the correct quadrant for both pairs using both historical CHARA ICCD speckle data and more recent United States Naval Observatory (USNO) ICCD speckle data. While preliminary analysis (Mason & Hartkopf 2002) generated long- and short-period solutions for both close pairs, the short-period, high-eccentricity solution has now been determined to be correct in both cases.

#### 4.4. New Old Measures

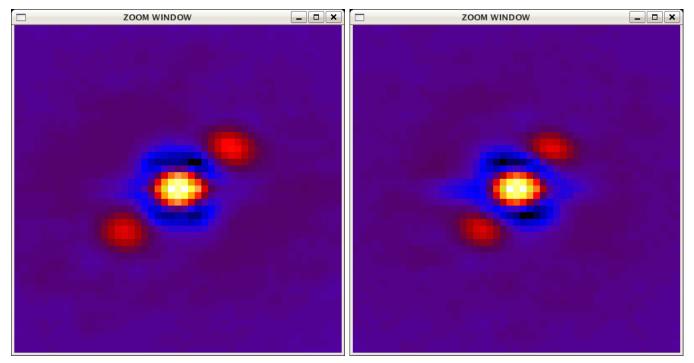
The first two measures taken with the CHARA CCD system were obtained at a relatively low magnification, such that both of the wider components were observed in the same dataset. As a result of their similar morphologies the closer subcomponents were found to overlap in Fourier space. Of the 13 peaks (n(n-1)+1) expected to be seen in autocorrelation space for a quadruple, only nine were seen. Figure 3(a) is the measured system geometry at the time of this observation and Figure 3(b) illustrates the resulting autocorrelogram. Figure 3(c) is the actual "full frame" directed vector autocorrelation (DVA) of the 1982

In mid-2007, Ellis Holdenried<sup>5</sup> developed software for calculating the DVA of a user-defined subarray of a CCD frame. Review of the archived videotape data, obtained in 1982 and dubbed in 1995, seemed to indicate that the tracking of the telescope was adequate and seeing was good, such that the selected subarray could be static rather than dynamic. While there was some degradation in the video signal, there is significant past experience in working with these old data and recovering good science (Hartkopf et al. 2000). Results of the application of the Holdenried subarray DVA for the two pairs are illustrated in Figure 4.

#### 4.5. New Measures

Additional observations made with the 4 m telescopes of Kitt Peak National Observatory and Cerro Tololo Interamerican Observatory were obtained with the USNO speckle camera in

<sup>&</sup>lt;sup>5</sup> USNO, retired.



**Figure 4.** Panel (a) is a DVA of the subarray around the brighter (Aa, Ab) pair of Finsen 332 while panel (b) is the fainter (Ba, Bb) pairing. These were generated from the same 1982.7650 data as shown in Figure 3(c). The images, at the same scale, provide a vivid representation of the pairs' similar morphologies. (A color version of this figure is available in the online journal.)

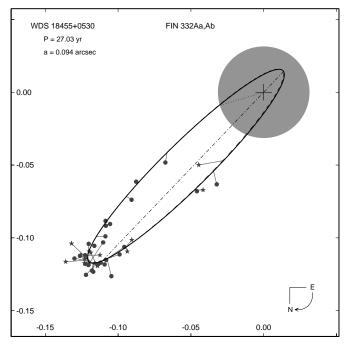
2001 and annually from 2005 to 2008. The system was also observed in 2006–2007 with the Mount Wilson 100 in telescope and in 2004 and 2008 with the Naval Observatory Flagstaff Station 61 inch telescope.

Also re-reduced was a 1996 April observation with the CHARA ICCD of Aa, Ab. The observation has been initially inspected with no measure obtained (Ba, Bb was published in Hartkopf et al. 2000). Reanalysis of the archived videotape allowed this measure, at quite a close separation, to be made; results are included in Table 1.

## 5. DISCUSSION

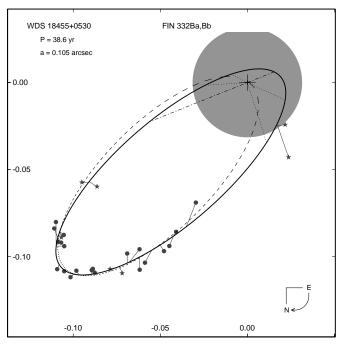
### 5.1. Orbit Determination

The larger errors associated with both micrometry and eyepiece interferometry, as well as the small  $\Delta m$  and the geometric peculiarities of the systems as illustrated in Figure 2, make them quite difficult to distinguish. However, observations by speckle interferometry are characterized by much lower errors (see Hartkopf et al. 2001). Therefore, only the measures obtained with 2 m or larger telescopes are utilized in the orbit analyses. The measures not included in the orbit determinations are indicated with notes in Table 1. The method of orbit calculation is the adaptive grid-search algorithm of Hartkopf et al. (1989), as modified by Mason et al. (1999). Briefly described, the Thiele-Innes elements (A, B, F, and G) are calculated via an iterative three-dimensional grid-search of elements P,  $T_o$ , and e with the search parameter space decreasing as the elements converge. The remaining Campbell elements (a'', i,  $\Omega$ , and  $\omega$ ) are then calculated directly. Observations are weighted using the scheme described in Hartkopf et al. (2001), which considers technique, observer expertise, the measured separation as a fraction of the telescope's Rayleigh limit, number of measures in a mean position, and any other notes the observer might have made with regard to quality. Table 4 gives the seven



**Figure 5.** Relative orbit of FIN 332Aa, Ab. The different techniques are represented by filled circles for published measures and filled stars for new or newly corrected measures. Only data used in this orbit determination are plotted. The measures are connected to the predicted position by an O-C line. The dashed line through the origin is the line of nodes. The light gray circle is the Rayleigh resolution limit  $(\frac{1.22\lambda}{D})$  of a 4 m telescope. Unresolved measures from 4 m class instruments are indicated by a dotted line drawn from the origin. The scale is in arcseconds and the direction of motion is indicated in the lower right corner. The barely distinguishable dashed curve is the short period solution of Mason & Hartkopf (2002).

orbital elements along with their associated errors for both Aa, Ab and Ba, Bb. The degree of success realized by the ensemble



**Figure 6.** Relative orbit of FIN 332Ba, Bb. Symbols are the same as Figure 5. Note the larger number of unresolved measures, and the greater divergence from the Mason & Hartkopf (2002) short period solution.

**Table 4**Orbital Elements of FIN 332Aa, Ab & Ba, Bb

Element	FIN 332Aa, Ab	FIN 332Ba, Bb
Period, P (yr)	$27.03 \pm 0.67$	$38.6 \pm 1.2$
Semimajor axis, $a''$	$0.094 \pm 0.019$	$0.105 \pm 0.015$
Inclination, <i>i</i> (°)	$106 \pm 20$	$117.2 \pm 9.5$
Longitude of node, $\Omega$ ( $^{\circ}$ )	$136.2 \pm 4.2$	$111.8 \pm 5.7$
Epoch of periastron, $T_o$ (yr)	$1994.20 \pm 0.98$	$1967.9 \pm 1.9$
Eccentricity, e	$0.79 \pm 0.34$	$0.867 \pm 0.034$
Longitude of periastron, $\omega$ ( $^{\circ}$ )	$10 \pm 16$	$311.2 \pm 8.3$
Grade	3	3

following the rubrics of Hartkopf et al. (2001), summarized by the grade, is also given here. Table 5 gives predicted positions ( $\rho$  and  $\theta$ ) at half-year intervals for the next 5 years. These orbits are illustrated in Figures 5 (Aa, Ab) and 6 (Ba, Bb).

## 5.1.1. Radial Velocity Measures

One of the items of greatest interest to investigate was the initial mention of radial velocity variability; however, this did not prove helpful in setting limits on orbital parameters. The spectral types of the components (A or a little later for each of them) make the measurement of radial velocity variability quite difficult due to the broad nature of the spectral features and the absence of many sharp metal lines. Plaskett et al. (1921) first noted variability and included STF2375 in their list of new spectroscopic binaries based on five observations obtained from 1920 June to October. Wilson (1953) added no new data in his catalog but gave it a quality rating of "acceptable." Palmer et al. (1968) added eight new velocities, but changed the mean by only 1 km s<sup>-1</sup>. Evans (1979) in his revision of Wilson's catalog later gave it a quality rating of "average."

While most of the components in the multiple system are broad-lined A stars, the Bb component may be an F star with sharper lines and it is possible that near periastron it may exhibit variable radial velocity features.

**Table 5** Ephemerides of FIN 332Aa, Ab & Ba, Bb

Epoch	FIN 332	2Aa, Ab	FIN 33	2Ba, Bb
	$\theta$	heta		$\rho$
	(°)	(")	(°)	(")
2010.0	312.0	0.160	6.8	0.043
2010.5	311.6	0.158	1.2	0.048
2011.0	311.2	0.155	356.6	0.053
2011.5	310.8	0.151	352.8	0.058
2012.0	310.3	0.147	349.6	0.063
2012.5	309.9	0.143	346.9	0.068
2013.0	309.4	0.139	344.5	0.072
2013.5	308.8	0.134	342.4	0.077
2014.0	308.3	0.128	340.5	0.081
2014.5	307.6	0.122	338.8	0.085

# 5.1.2. Interpreting Unresolved Measures

As seen in Tables 1 and 2 and illustrated in Figure 2, there are two recent times in the short-period orbits when the pairs were predicted to be unresolved: Aa, Ab from 1964.6 to 1969 and again from 1991.7 to 1996.1, Ba, Bb from 1966.7 to 1972.1 and again from 2005.2 to 2010.7. The later two periods of predicted non-resolution corresponded to multiple null detections for both pairs, as indicated in Tables 1 and 2. These non-resolutions, while not utilized in determining these orbits, are completely consistent with the solutions.

#### 5.2. Mutual Inclination

FIN 332 offers the rare possibility of determining the mutual inclination of orbits in a quadruple system whose subsystems are at the same hierarchical level. A first glance shows that the individual orbital inclinations agree to with  $1\sigma$ . However, the mutual inclination of their orbital planes is also dependent upon their nodal longitudes as given in the relation:

$$\cos(\phi) = \cos(i_{Aa,Ab})\cos(i_{Ba,Bb}) + \sin(i_{Aa,Ab})\sin(i_{Ba,Bb})$$
$$\times \cos(\Omega_{Aa,Ab} - \Omega_{Ba,Bb}).$$

Inserting the values of  $(i,\Omega)_{Aa,Ab}$  and  $(i,\Omega)_{Ba,Bb}$  from Table 4 into this relation yields a mutual inclination of  $\phi_{AB}=25^{\circ}.2\pm12^{\circ}.2$ . This indicates that the two orbits are more coplanar than not; however, if we adopt the threshold for coplanarity defined by Fekel (1981) of  $\phi<15^{\circ}$  then these two orbits are within  $1\sigma$  of being coplanar.

We have thus far assumed that the nodes specified by  $\Omega_{Aa,Ab}$  and  $\Omega_{Ba,Bb}$  are indeed the ascending nodes, but, regrettably, there is no spectroscopy to support that assumption. The two orbital inclinations reflect that both orbits are revolving in a retrograde sense, i.e., their P.A.s are decreasing with time. Interestingly, the wide and very long-period system is clearly moving in a direct sense with P.A.s increasing with time.

Because of the degeneracy of the Omegas, a second possible value for the mutual inclination of  $49^\circ.3 \pm 19^\circ.6$  arises. While the eye is naturally attracted to the case of identical nodal quadrants, without radial velocity verification, which has already been shown to be a very challenging task, there remains the possibility that nature lacks the aesthetic of the eye. At this point, all we can state is that while the two orbital planes may be nearly coplanar, they are most certainly not nearly perpendicular.

### 5.3. Mass Sums

While both Aa and Ba are listed as spectral type A1V in the Multiple Star Catalogue (Tokovinin 1997), the spectral types

of the secondaries are not known. Given the small magnitude differences (discounting the Hipparcos  $\Delta m$ ) it is conceivable that we have four A dwarf stars with expected mass sums of each pair between 5 and 6  $\mathcal{M}_{\odot}$ . Unfortunately, given the large errors in the parallax and orbital elements these are of little help. The Aa, Ab solution gives a mass sum of  $12 \pm 16$   $\mathcal{M}_{\odot}$  while that of Ba, Bb is  $7.6 \pm 9.0$   $\mathcal{M}_{\odot}$ . While their orbital elements can undoubtedly be improved, especially if they are resolved during periastron, the largest improvement may come from a more precise determination of their parallax.

## 5.4. Stitching "Dum" and "Dee" on Their Collars

For the first several decades since their discovery, the peculiar geometries of these systems made them nearly indistinguishable. If we compare their predicted position and subjectively qualify them as "similar" when their positions are approximately the same:  $d\theta < 10^\circ$  and  $d\rho < 0''.05$  or both < 0''.05, i.e., unresolved, they would be qualified as "similar" for 33% of the next thousand years. While their appearances have diverged somewhat in recent years, by the middle of this century both pairs will again go through periastron within a few years of each other and FIN 332 Aa, Ab and Ba, Bb will again exemplify their Carrollian sobriquets.

We acknowledge W. S. Finsen and W. van den Bos for their pioneering work in double star interferometry in the southern hemisphere. Also we thank the USNO on-site archives for access to the voluminous correspondence between W. S. Finsen and C. E. Worley, head of the USNO visual double star program and Washington Double Star catalog projects for more than 30 years. The USNO speckle interferometry program has been supported by the National Aeronautics and Space Administration under reimbursable NNH06AD70I, issued through the Terrestrial Planet Finder Foundation Science program. This research has made use of the SIMBAD database, operated at CDS, Strasbourg, France. Thanks are also extended to Ken Johnston and the U. S. Naval Observatory for their continued support of the Double Star Program.

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# MASON, HARTKOPF, & MCALISTER

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